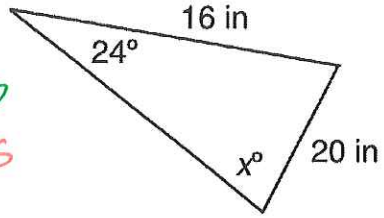
Skills Practice 7.2.1: Law of Sines and Law of Cosines

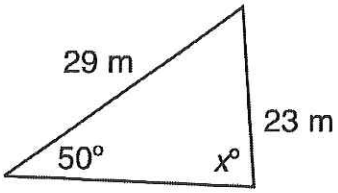
Use the law of sines or the law of cosines to solve for x, to the nearest tenth, in each triangle. If using the law of sines to determine an angle measure, include all possible angle measures.

1. 

AAS
Sines

$\frac{\sin x}{16} = \frac{\sin 24}{20}$ $x = \sin^{-1}\left(\frac{16\sin 24}{20}\right)$

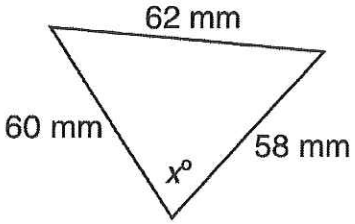
$x = 19°$

2. 

ASS
Sines

$\frac{\sin x}{29} = \frac{\sin 50}{23}$ $x = \sin^{-1}\left(\frac{29\sin 50}{23}\right)$

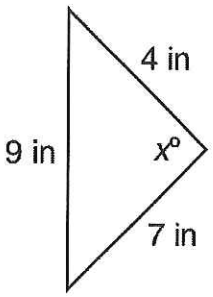
$x = 75°$

3. 

SSS
Cosines

$\cos X = \frac{58^2 + 60^2 - 62^2}{2(58)(60)}$ $x = \cos^{-1}\left(\frac{3120}{6960}\right)$

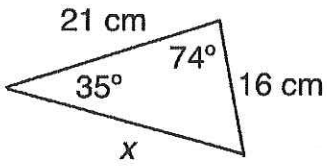
$x = 63°$

4. 

SSS
Cosines

$\cos x = \frac{4^2 + 7^2 - 9^2}{2(4)(7)}$ $x = \cos^{-1}\left(\frac{-16}{56}\right) = 107$

$x = 107°$

5. 

AAS or ASA
Sines

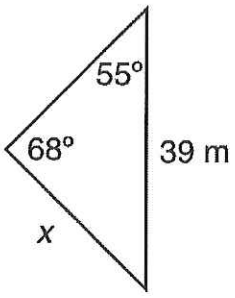
$\frac{x}{\sin 74} = \frac{16}{\sin 35}$

$x = 26.8$

$x = \frac{16\sin 74}{\sin 35} = 26.8$

continued

6.

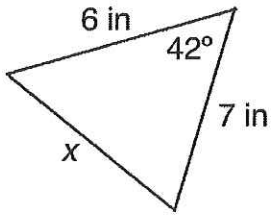


AAS
Sines

$\frac{x}{\sin 55^\circ} = \frac{39}{\sin 68^\circ}$

$x = \frac{39 \sin 55}{\sin 68} = 34.5 = x$

7.

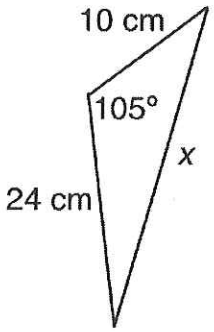


SAS
Cosines

$x = \sqrt{6^2 + 7^2 - 2(6)(7)\cos 42}$

$x = 4.8$

8.

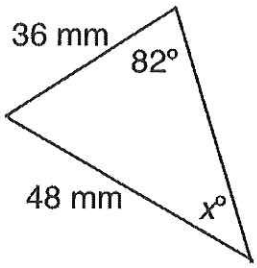


SAS
Cosines

$x = \sqrt{24^2 + 10^2 - 2(24)(10)\cos 105}$

$x = 28.3$

9.

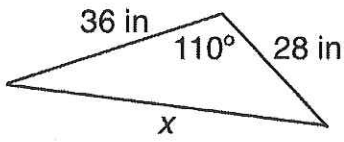


ASS
Sines

$\frac{\sin x}{36} = \frac{\sin 82}{48}$

$x = \sin^{-1}\left(\frac{36 \sin 82}{48}\right) = 48^\circ = x$

10.



SAS
Cosines

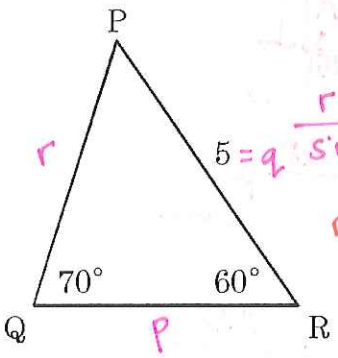
$x = \sqrt{28^2 + 36^2 - 2(28)(36)\cos 110}$

$x = 52.6$

Law of Sines Practice

Name: Answer Key **Date:** _____

1. In the triangle below, what is the measure of $\overline{PQ}$?

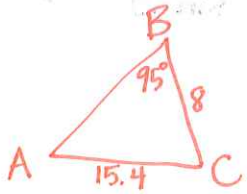


$\frac{r}{\sin 60} = \frac{5}{\sin 70}$

$r = \frac{5\sin 60}{\sin 70}$

A. $\frac{5}{\sin 70°}$ B. $\frac{5}{\sin 70°}$

C. $\frac{5\sin 60°}{\sin 70°}$ D. $\frac{5}{\sin 60°\sin 70°}$



$\frac{\sin A}{8} = \frac{\sin 95}{15.4}$

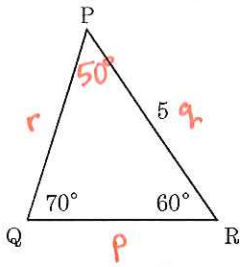
2. In $\triangle ABC$, find $\angle A$ to the nearest tenth of a degree if $m\angle B = 95°$, $b = 15.4$, and $a = 8.0$.

A. 31.2° B. 32.1° C. 84.0° D. 92.3°

$A = \sin^{-1}\left(\frac{8\sin 95}{15.4}\right)$

$= 31.16496...$

3. In the triangle, what is the measure of $\overline{QR}$?



A. $\frac{5}{\sin 50°}$

B. $\frac{5\sin 50°}{\sin 70°}$

C. $\frac{5}{\sin 70°}$

D. $\frac{5}{\sin 50°\sin 70°}$

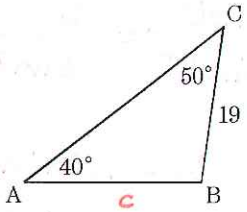
$\frac{p}{\sin 50°} = \frac{5}{\sin 70}$

$p = \frac{5\sin 50}{\sin 70}$

4. To the nearest tenth of a centimeter, what is the length of side AB?

A. 22.6 B. 24.8

C. 28.5 D. 35.6



$\frac{c}{\sin 50} = \frac{19}{\sin 40}$

$c = \frac{19\sin 50}{\sin 40} = 22.6$

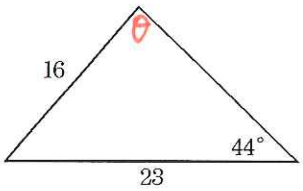
5. To the nearest degree, what is the measure of the *largest* angle of the triangle?

A. 32

B. 62

C. 87

D. 103

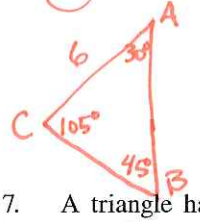


$\frac{\sin\theta}{23} = \frac{\sin 44}{16}$

$\theta = \sin^{-1}\left(\frac{23\sin 44}{16}\right) = 86.937$

6. In $\triangle ABC$, $A = 30°$, $C = 105°$ and $b = 6$. Find a.

A. $3\sqrt{2}$ B. $6\sqrt{2}$ C. $\sqrt{2}$ D. $2\sqrt{2}$



$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b\sin A}{\sin B} = a = \frac{6\sin 30}{\sin 45} = 4.24$

7. A triangle has sides of lengths a, b, and c, then, according to the sine law, what does a equal?

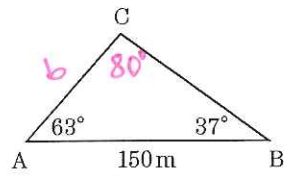
A. $\frac{c\sin B}{\sin C}$ B. $\frac{C\sin A}{\sin B}$

C. $\frac{b\sin C}{\sin A}$ D. $\frac{b\sin A}{\sin B}$

$\sin A \cdot \frac{a}{\sin A} = \frac{b}{\sin B} \cdot \sin A$

$a = \frac{b\sin A}{\sin B}$ OR $a = \frac{c\sin A}{\sin C}$

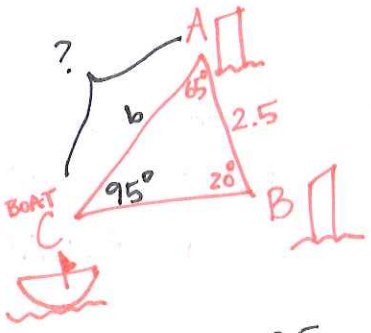
8. To 1 decimal place, what is the length of $\overline{AC}$?



$$\frac{150}{\sin 80} = \frac{b}{\sin 37}$$

$$b = \frac{150 \sin 37}{\sin 80} = 91.7 = \overline{AC}$$

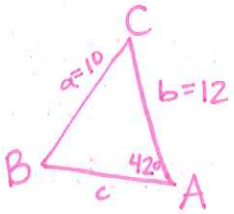
9. Two lighthouses (at A and B) on a relatively straight shore are 2.5 km apart. They both spot a boat at point C. If $m\angle BAC = 65°$ and $m\angle CBA = 20°$, then how far is the boat from the lighthouse A? Answer to 2 decimal places.



$$\frac{b}{\sin 20°} = \frac{2.5}{\sin 95°}$$

$$b = \frac{2.5 \sin 20°}{\sin 95°} = .85 \text{ km}$$

10. In acute triangle ABC, side $a = 10$, side $b = 12$, and $m\angle A = 42$. Find $m\angle B$ to the nearest degree.



$$\frac{\sin 42°}{10} = \frac{\sin B}{12}$$

$$\sin^{-1}\left(\frac{12 \sin 42°}{10}\right) = B$$

$$B = 53°$$

Law of Cosines Practice

Name: Answer Key **Date:** _____

DEGREES

1. In $\triangle ABC$, $a = 3$, $b = 5$, and $m\angle C = 120°$. Find the value of c.

$c^2 = a^2 + b^2 - 2ab\cos C$

$c = \sqrt{9 + 25 - 2(3)(5)\cos 120}$

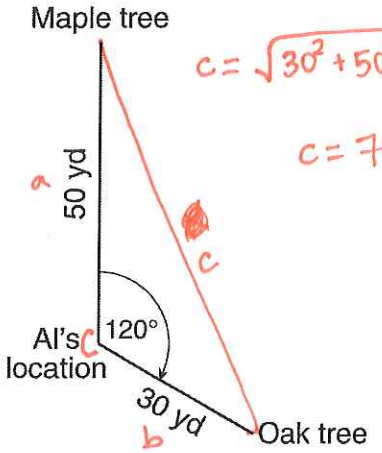
$c = 7$

2. In triangle ABC, $a = 5$, $b = 8$, and $m\angle C = 60$. Find the length of side c.

$c = \sqrt{25 + 64 - 2(5)(8)\cos 60}$

$c = 7$

3. Al is standing 50 yards from a maple tree and 30 yards from an oak tree in the park. His position is shown in the accompanying diagram. If he is looking at the maple tree, he needs to turn his head 120° to look at the oak tree.



$c = \sqrt{30^2 + 50^2 - 2(30)(50)\cos 120}$

$c = 70$

How many yards apart are the two trees?

A. 58.3 B. 65.2 (C.) 70 D. 75

4. In a triangle, the sides measure 3, 5, and 7. What is the measure, in degrees, of the largest angle? (a, b, c)

A. 60 B. 90 (C.) 120 D. 150

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $\cos^{-1}\left(\frac{-1}{2}\right) = 120°$

$\cos C = \frac{3^2 + 5^2 - 7^2}{2(3)(5)} = \frac{-15}{30}$

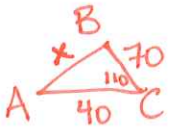
5. In $\triangle DEF$, if side $d = 14$, side $e = 10$, and side $f = 12$, find $m\angle F$ to the *nearest degree*.

$\cos F = \frac{d^2 + e^2 - f^2}{2de}$ $F = \cos^{-1}\left(\frac{152}{280}\right)$

$\cos F = \frac{14^2 + 10^2 - 12^2}{2(10)(14)} = \frac{152}{280}$ $F = 57°$

6. The distance from A to C is 40 miles and the distance from C to B is 70 miles. If $m\angle ACB = 110$, find AB to the *nearest mile*. [*Show or explain the procedure used to obtain your answer.*]

$\overline{AB} = \sqrt{40^2 + 70^2 - 2(40)(70)\cos 110}$



$= 92$ miles

7. At a point 8 miles from one mountain peak and 3 miles from another, the angle between them is 65°. To the nearest tenth of a mile, how far apart are the mountain peaks?

A. 6.9 miles B. 7.7 miles

C. 7.0 miles (D.) 7.3 miles

$\sqrt{3^2 + 8^2 - 2(3)(8)\cos 65}$

$= 7.3$

8. Given $\triangle ABC$ with sides $a = 10$, $b = 12$ and $c = 15$ units, the sum of angles A and B, correct to the nearest hundreth is:

A. 85.46°. (B.) 94.54°.

C. 102.67°. D. 86.54°.

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$= \frac{10^2 + 15^2 - 12^2}{2(10)(15)}$

$= \frac{181}{300}$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos^{-1}\left(\frac{269}{360}\right) = 41.65°$

$= \frac{12^2 + 15^2 - 10^2}{2(12)(15)} = \frac{269}{360}$ $\cos^{-1}\left(\frac{181}{300}\right) = 52.89$

9. The smallest angle in a right triangle with sides, 28, 45, and 53 is approximately:

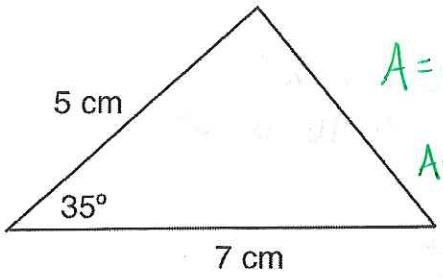
A. 27.8° (B.) 31.9° C. 51.5° D. 58.1°

$\cos A = \frac{45^2 + 53^2 - 28^2}{2(45)(53)}$ $A = \cos^{-1}\left(\frac{405}{477}\right)$

$= \frac{4050}{4770}$ $A = 31.9°$

Skills Practice 7.2.2: Area of Triangles

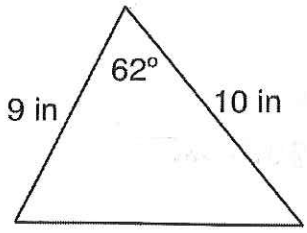
Find the area, to the nearest tenth, of each triangle. Draw a diagram if needed.

1. 

$A = \frac{1}{2}ab\sin C$

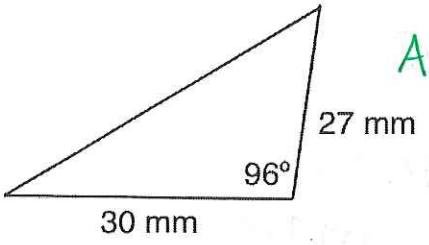
$\text{Area} = \frac{1}{2}(5)(7)\sin 35°$

$\text{Area} = 10.0 \text{ cm}^2$

2. 

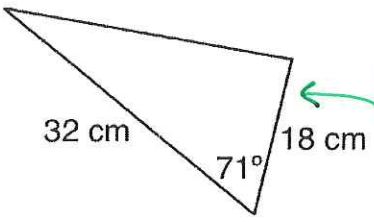
$A = \frac{1}{2}(9)(10)\sin 62°$

$\text{Area} = 39.7 \text{ in}^2$

3. 

$A = \frac{1}{2}(27)(30)\sin 96°$

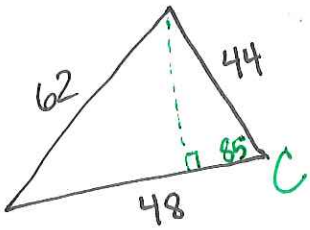
$\text{Area} = 402.8 \text{ mm}^2$

4. 

$A = \frac{1}{2}(18)(32)\sin 71°$

$\text{Area} = 272.3 \text{ cm}^2$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$



$\cos C = \frac{44^2+48^2-62^2}{2(44)(48)}$

$\cos C = \frac{396}{4224}$

$C = \cos^{-1}\left(\frac{396}{4224}\right) = 85°$

Law of Cosines to find an angle.

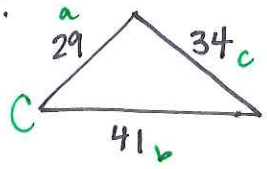
5. Side lengths: 44 in., 48 in., 62 in. (a, b, c)
6. Side lengths: 29 mm, 41 mm, 34 mm
7. Side lengths: 50 cm, 40 cm, 72 cm
8. Side lengths: 19 cm, 16 cm, 10 cm
9. Side lengths: 54 in., 62 in., 58 in.
10. Side lengths: 36 mm, 39 mm, 43 mm

[#6-10 on back]

$A = \frac{1}{2}ab\sin C$

$\text{Area} = \frac{1}{2}(44)(48)\sin 85°$

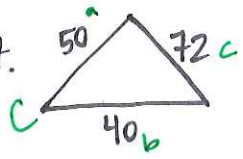
#5 $\text{Area} = 1051.35 \text{ in}^2$

6. 

$C = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right) = 55°$

$\text{Area} = \frac{1}{2}ab\sin C \approx 486.6\ mm^2$

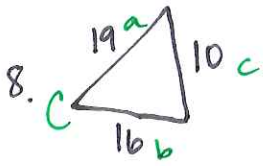
OR 487.0 mm^2

7. 

$C = \cos^{-1}\left(\frac{50^2+40^2-72^2}{2(50)(40)}\right) = 106°$

$\text{Area} = \frac{1}{2}(50)(40)\sin 106° \approx 962.6\ cm^2$

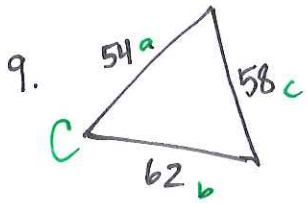
OR 961.3 cm^2

8. 

$C = \cos^{-1}\left(\frac{19^2+16^2-10^2}{2(19)(16)}\right) = 32°$

$\text{Area} = \frac{1}{2}(19)(16)\sin 32° \approx 80.0\ cm^2$

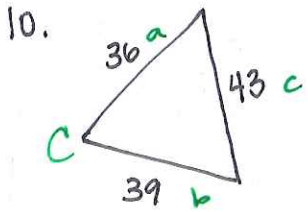
OR 80.5 cm^2

9. 

$C = \cos^{-1}\left(\frac{54^2+62^2-58^2}{2(54)(62)}\right) = 60°$

$\text{Area} = \frac{1}{2}(54)(62)\sin 60° \approx 1442.7\ in^2$

OR 1449.7 in^2

10. 

$C = \cos^{-1}\left(\frac{36^2+39^2-43^2}{2(36)(39)}\right) = 70°$

$\text{Area} = \frac{1}{2}(36)(39)\sin 70° = 659.0\ mm^2$

OR 659.7 mm^2